Computational and Algorithmic Thinking

CAT Sample Problems
Senior

AUSTRALIAN MATHS TRUST

COMPUTATIONAL AND ALGORITHMIC THINKING

Part A: Question 3

This question should be answered by a single choice from A to E.
This question is worth 3 mark.

3. Bubble Up

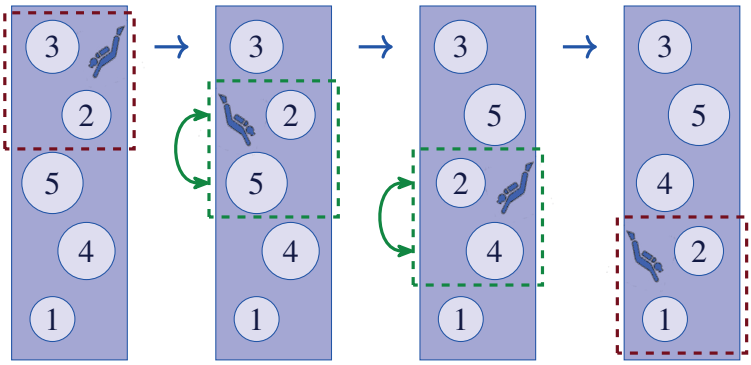
Bubble Up is a mobile phone game where players need to put a column of underwater bubbles in order of size.

During a game the player makes several 'dives'. Each dive starts from the top and goes down to the bottom. At each bubble during a dive, the player makes a choice to:

- skip over this bubble and go down to the next one, *OR*
- swap this bubble with the bubble that is *just below* it.

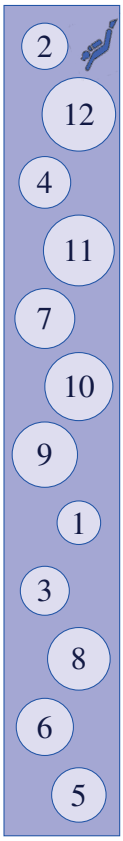
Example

There are five bubbles at the start of a dive. They appear in the order 32541. The player decides to leave the first two as they are, then swap the 2 and 5, then swap the 2 and 4, then leave the bottom two alone. So in this one dive, the player has reordered the bubbles to make 35421. Another dive will be required to complete the game.



Remember the player can never move upwards during a single dive.

Shown here is the starting position for a more advanced level of Bubble Up.



What is the smallest number of dives required to get all the bubbles in order with the 12 at the top and the 1 at the bottom?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Part B: Question 7

This question has three parts, each of which is worth 2 marks.
Each part should be answered by a number in the range 0–999.

7. Race Track

Alain races his car in time trials on race tracks made up of straights and corners. Each corner has a speed limit, and if he enters the corner any faster than the limit, he crashes.

The straights are laid out in segments, marked by lines across the track.

On each straight segment he can do one of the following:

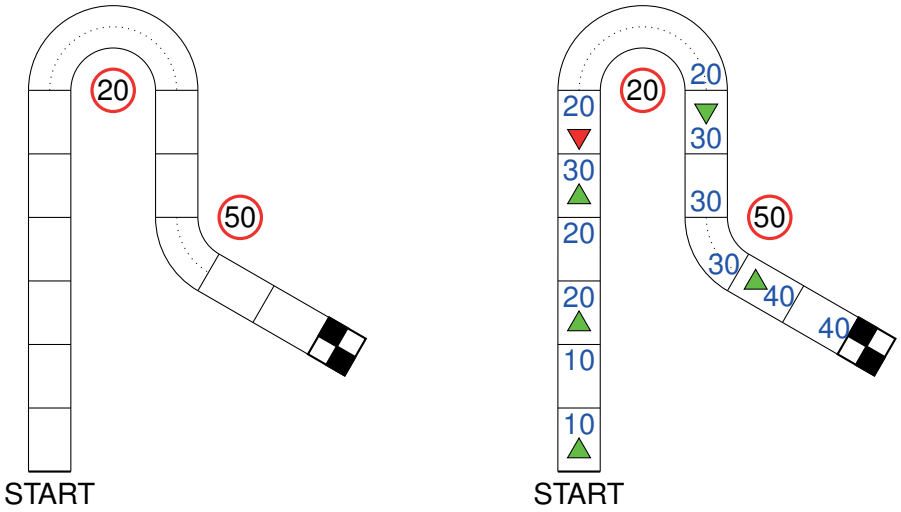
- accelerate to increase his speed by 10km/h
- brake to decrease his speed by 10km/h
- coast, which keeps his speed the same.

In corners, his car cannot accelerate or brake, and so exits the corner at the same speed that it enters – unless he is over the corner's speed limit, when he will crash.

His car always starts at the start line and has zero speed when the time trial starts. It can safely cross the finish line at any speed.

Example

For example, while testing some experimental tyres on the test track on the left, Alain drove as illustrated on the right.



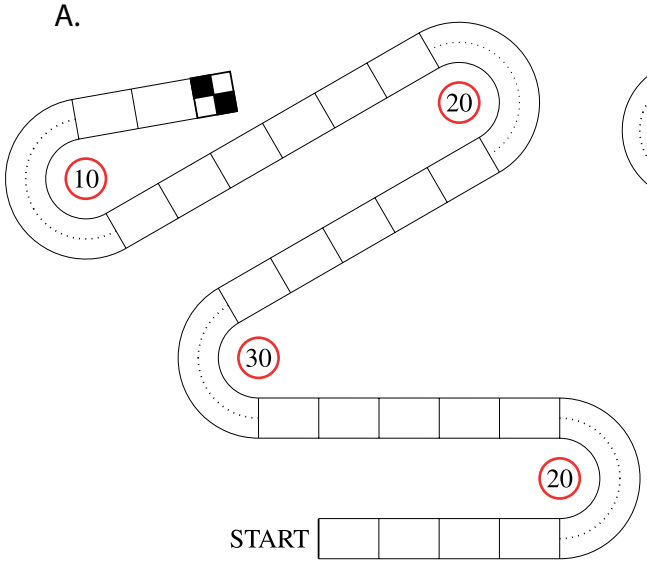
A faster drive than this is possible, but he can't go so fast that he crashes. For example, if he had accelerated on every segment of the first straight, he would have entered the first corner at 60km/h and crashed.

Each time Alain accelerates, he uses $10 worth of the team's fuel, and each time he brakes it costs the team $1 in tyre wear. In the test drive above the total cost was $51.

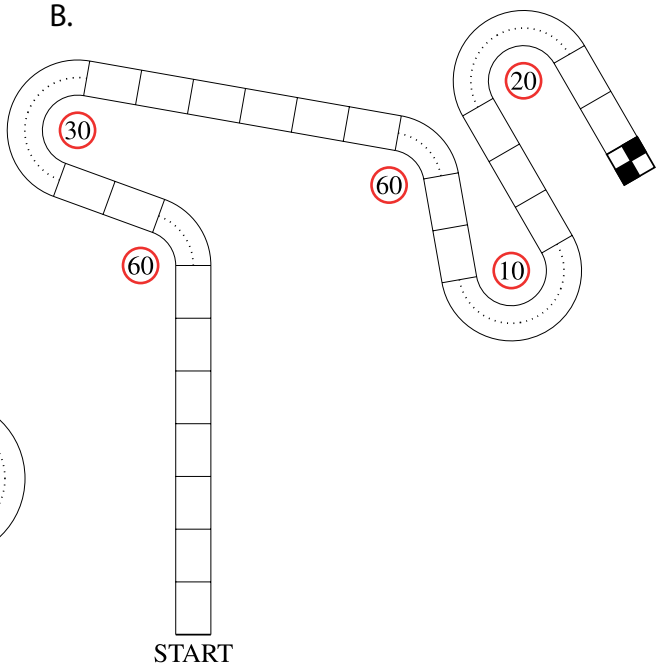
In the competition time trials, *Alain always drives as fast as he possibly can on each segment* without crashing.

On each of the following courses, what is the total cost his team incurs when Alain races once?

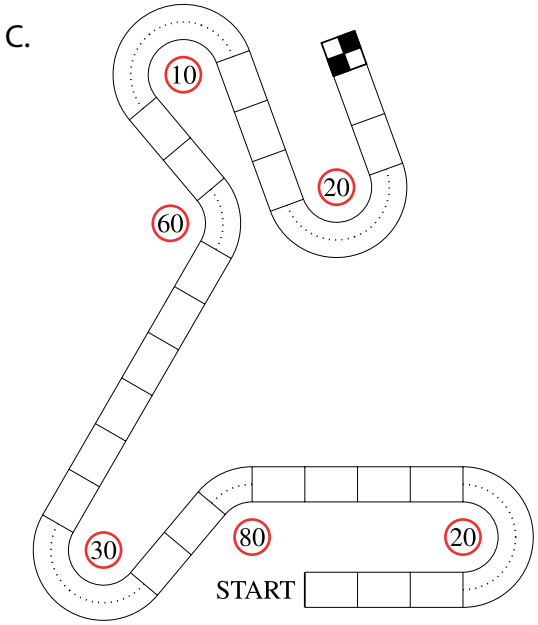
A.



B.



C.



Part C: Prize Question 1

This prize question is not part of the core competition. The prize question has two parts and has the same rules as a core question in the paper.

Note: the full introductory text is copied below but the examples are omitted.

Prize 1. Race Track

Alain races his car in time trials on race tracks made up of straights and corners. Each corner has a speed limit, and if he enters the corner any faster than the limit, he crashes.

The straights are laid out in segments, marked by lines across the track.

On each straight segment he can do one of the following:

- accelerate to increase his speed by 10km/h
- brake to decrease his speed by 10km/h
- coast, which keeps his speed the same.

In corners, his car cannot accelerate or brake, and so exits the corner at the same speed that it enters – unless he is over the corner's speed limit, when he will crash.

His car always starts at the start line and has zero speed when the time trial starts. It can safely cross the finish line at any speed.

Prize Extensions

In these questions we represent a racetrack by s_1 (C_1) s_2 ...
The racetrack in the example would be represented by 6 (20) 2 (50) 2.

Alain is racing several laps on a racetrack where the end point is the same as the starting point. Alain still drives as fast as he possibly can on each segment without crashing.

In these questions, when Alain accelerates his speed only increases by 1 (one) on each segment.

A. The race track is 4 (61) 4 (61) 4 (60) 4 (54) 4.
Alain drives 4 times around the track. On how may segments did he brake?

B. The race track is 5 (53) 5 (54) 5 (49) 5 (44) 5.
Alain intends to drive 3 times around the track. Unfortunately his brakes are not working. He does not know this, and drives as if they were. This causes him to crash.
How fast was he travelling when he crashed?