Computational and Algorithmic Thinking

CAT Sample Solutions

AUSTRALIAN MATHS TRUST

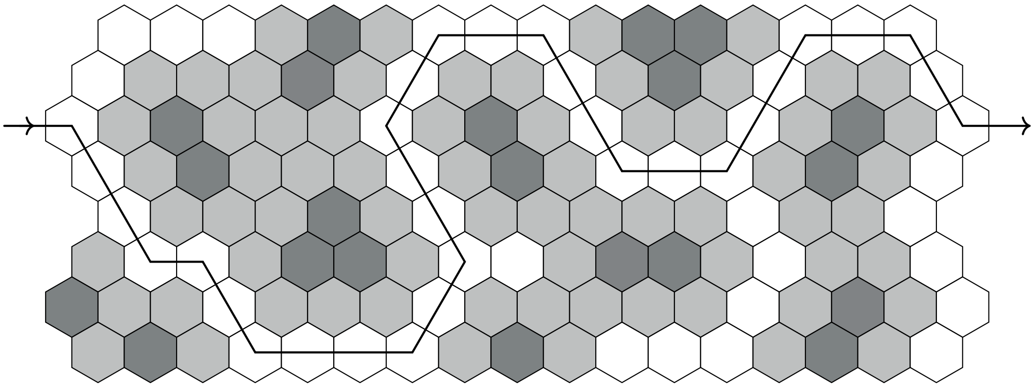
COMPUTATIONAL AND ALGORITHMIC THINKING

Upper Primary Solutions

Part A: Question 1

1. Drop Bears

In the diagram below, the lightly shaded hexagons contain trees next to a tree with a drop bear.



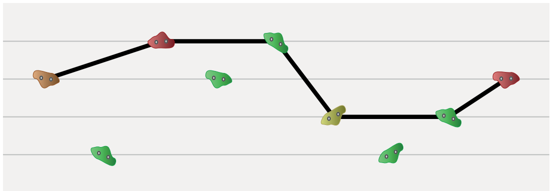
The line shows Juliet's route. She will walk under 31 trees.
Hence (E).

Part B: Question 7

7. Climbing Wall Challenge

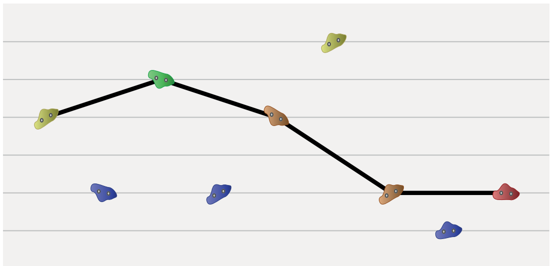
Our aim here is to make differences in height as small as possible.
The dark lines show the route taken.

A.



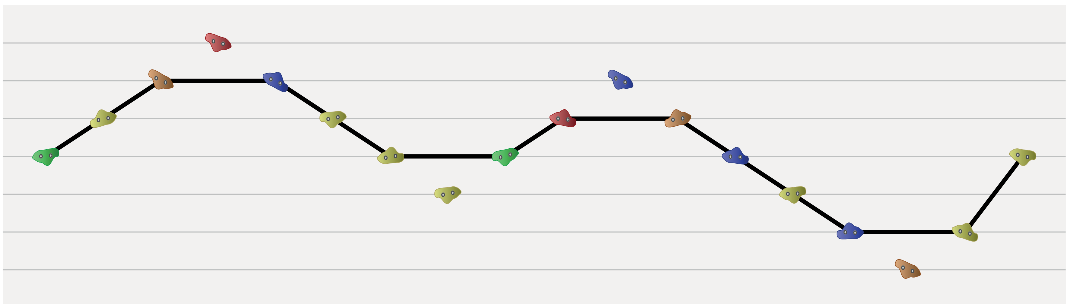
The extra time in increasing and decreasing height $= 2 + 0 + 2 + 0 + 2 = 6$ seconds.

B.



The extra time in increasing and decreasing height $= 2 + 1 + 2 + 0 = 5$ seconds.

C.



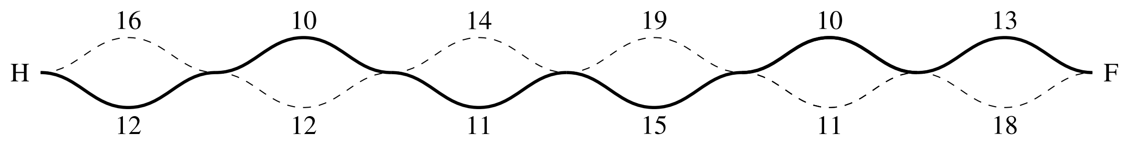
The extra time in increasing and decreasing height

$= 4 + 0 + 2 + 0 + 2 + 0 + 2 + 1 + 0 + 4 = 15$ seconds.

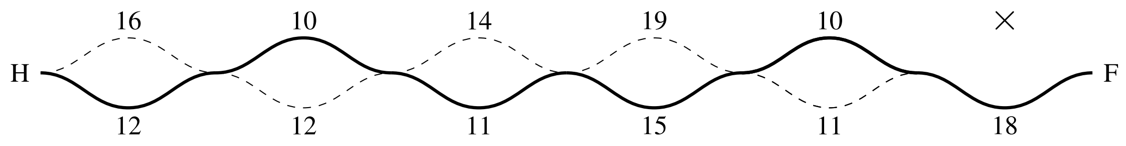
Junior Solutions
Question 3

3. Landslide

Before the landslide, Ketura took the shorter of the two sections on each leg.



The distance she had to travel was $12+10+11+15+10+13=71$.
The extra distance is $76-71=5$.
This is the difference between the two sections on the last leg.

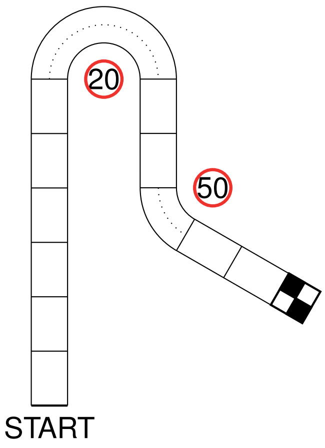


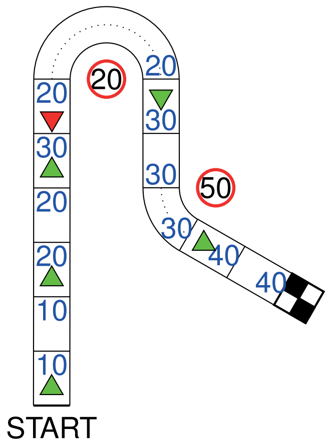
The landslide closed the shorter section on the last leg, which is 13.
Hence (D).

Question 8

8. Race Track

We will linearise the maps and use $+$ for accelerate, $-$ for brake and $\backsim$ for coast in the drives. Here is this notation applied to the example.





start 6 [20] 2 [50] 2 *end* $\qquad$ *start* $+\backsim+\backsim+-$ [20] $+\backsim$ [50] $+\backsim$ *end*

When we know Alain's speed on each corner, his drive on each straight can be determined independently. Along a straight, his speed on each segment is greatest if he accelerates before cruising or braking, and cruises before braking.

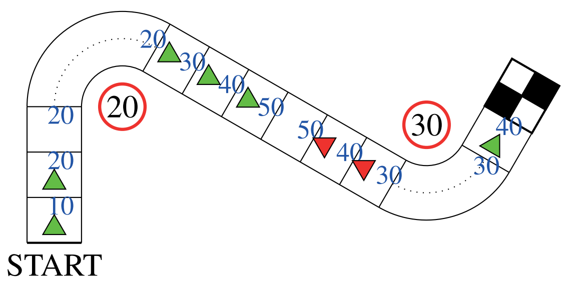
For tracks A and B, there are enough segments on the straight leading into a corner for him to take it at its limit.

A.

start 3 [20] 6 [30] 1 *end*

Straight	Drive
start 3 [20]	$++\backsim$
[20] 6 [30]	$+++\backsim--$
[30] 1 *end*	$+$

Cost $= 6\times 10+2 = 62$.

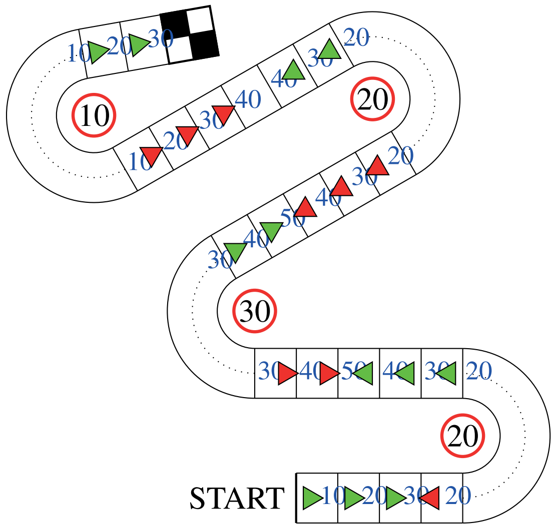


B.

start 4 [20] 5 [30] 5 [20] 6 [10] 2 *end*

Straight	Drive
start 4 [20]	$+++-$
[20] 5 [30]	$+++--$
[30] 5 [20]	$++---$
[20] 6 [10]	$++\backsim---$
[10] 2 *end*	$++$

Cost $= 12 \times 10 + 9 = 129$.



C.

start 7 [60] 2 [30] 6 [60] 2 [10] 3 [20] 2 *end*

Here there are not enough segments to slow from 60 to 30 in the straight [60] 2 [30].

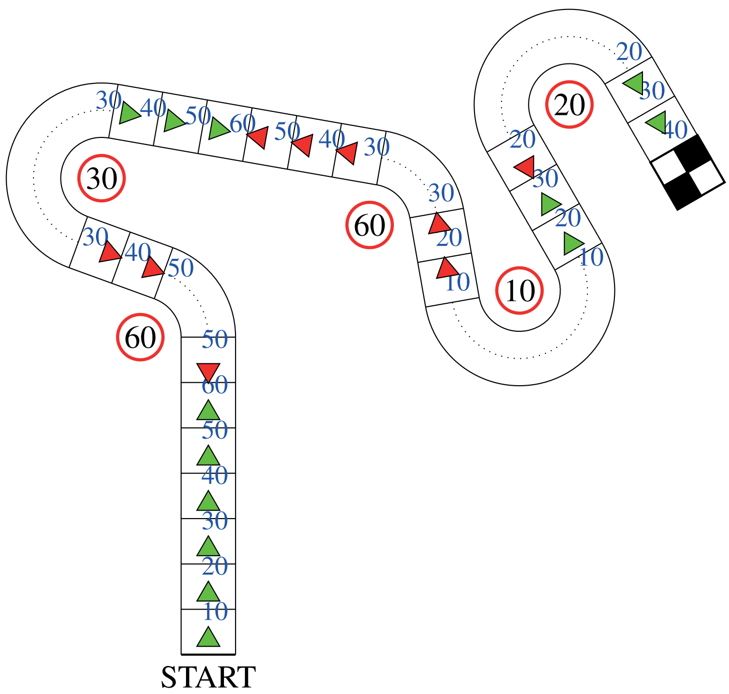
We address this by reducing the 60 to 50, as 2 segments is enough to slow from 50 to 30. The straight becomes [50] 2 [30].

Similarly the straight [60] 2 [10] becomes [30] 2 [10].

The drive now becomes *start* 7 [50] 2 [30] 6 [30] 2 [10] 3 [20] 2 *end* and we can treat it as above.

Straight	Drive
start 7 [50]	$++++++-$
[50] 2 [30]	$--$
[30] 6 [30]	$+++---$
[30] 2 [10]	$--$
[10] 3 [20]	$++-$
[20] 2 *end*	$++$

Cost $= 13 \times 10 + 9 = 139$.

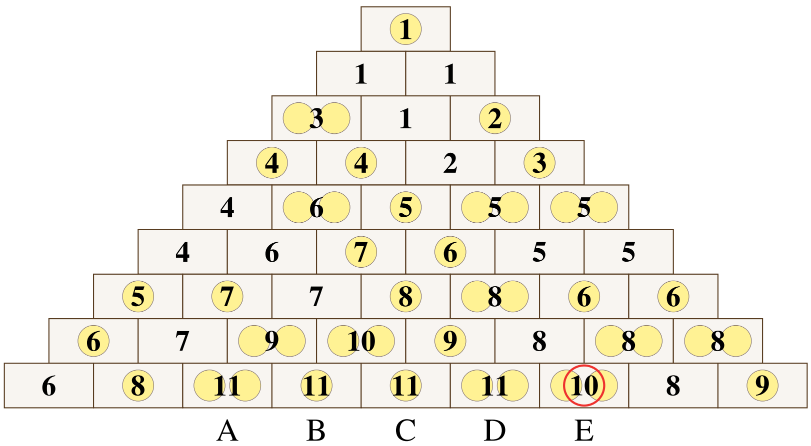


Intermediate Solutions
Part A: Question 1

1. Pyramid Climb

The key to solving this problem efficiently is to reverse the journey – start at the top and work your way down. For a given brick on the bottom row, the best possible bottom-to-top path that starts there is the same as the best possible top-to-bottom path that finishes there.

In the following diagram, the number in each brick indicates the maximum number of coins that could be collected on your way there starting from the top. To find these subtotals, take the larger of the two numbers above (or the one number above if on the edge) and add the number of coins in that brick.

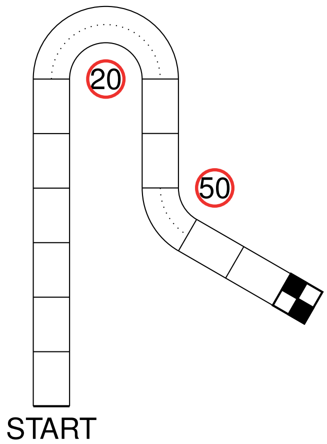


Among bricks A to E, the worst place to start is brick E with a maximum of 10 coins. Hence (E).

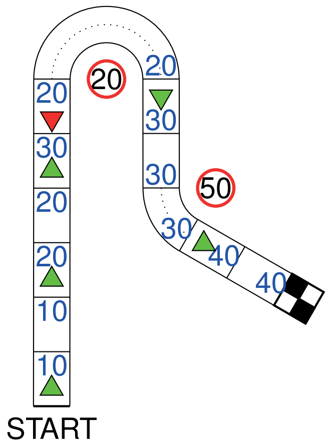
Part B: Question 7

7. Race Track

We will linearise the maps and use $+$ for accelerate, $-$ for brake and $\backsim$ for coast in the drives. Here is this notation applied to the example.



start 6 [20] 2 [50] 2 *end*



start $+ \backsim + \backsim + -$ [20] $+ \backsim$ [50] $+ \backsim$ *end*

When we know Alain's speed on each corner, his drive on each straight can be determined independently. Along a straight, his speed on each segment is greatest if he accelerates before cruising or braking, and cruises before braking.

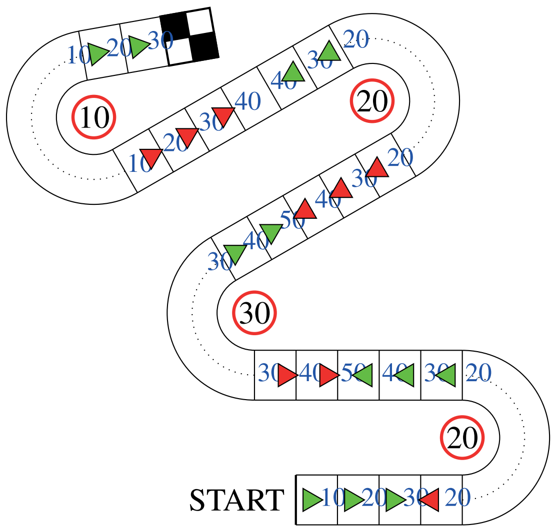
For track A, there are enough segments on the straight leading into a corner for him to take it at its limit.

A.

start 4 [20] 5 [30] 5 [20] 6 [10] 2 *end*

Straight	Drive
start 4 [20]	$+++-$
[20] 5 [30]	$+++--$
[30] 5 [20]	$++---$
[20] 6 [10]	$++\backsim---$
[10] 2 *end*	$++$

Cost $= 12 \times 10 + 9 = 129$.



B.

start 7 [60] 2 [30] 6 [60] 2 [10] 3 [20] 2 *end*

Here there are not enough segments to slow from 60 to 30 in the straight [60] 2 [30].

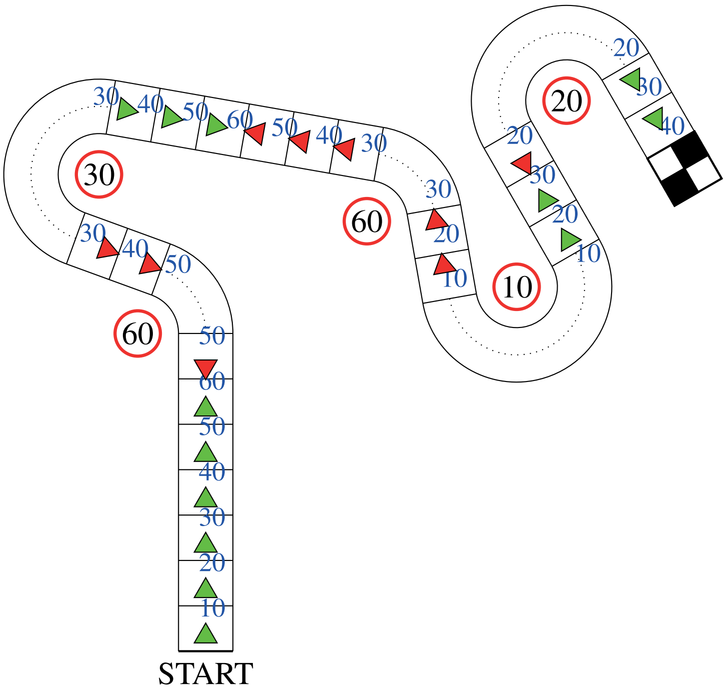
We address this by reducing the 60 to 50, as 2 segments is enough to slow from 50 to 30. The straight becomes [50] 2 [30].

Similarly the straight [60] 2 [10] becomes [30] 2 [10].

The drive now becomes *start* 7 [50] 2 [30] 6 [30] 2 [10] 3 [20] 2 *end* and we can treat it as above.

Straight	Drive
start 7 [50]	+ + + + + + + −
[50] 2 [30]	− −
[30] 6 [30]	+ + + − − −
[30] 2 [10]	− −
[10] 3 [20]	+ + −
[20] 2 *end*	+ +

Cost $= 13 \times 10 + 9 = 139$.



C.

start 3 [20] 4 [80] 2 [30] 6 [60] 2 [10] 3 [20] 2 *end*

On this track we have two problems. Firstly there are not enough segments on the second straight to reach the limit on the second corner.

Here we address this by reducing the speed on the second corner to 70.

The drive then becomes *start* 3 [20] 4 [70] 2 [30] 6 [60] 2 [10] 3 [20] 2 *end*.

Secondly there are not enough segments to slow from 70 to 30 in the third straight or from 60 to 10 in the fifth.

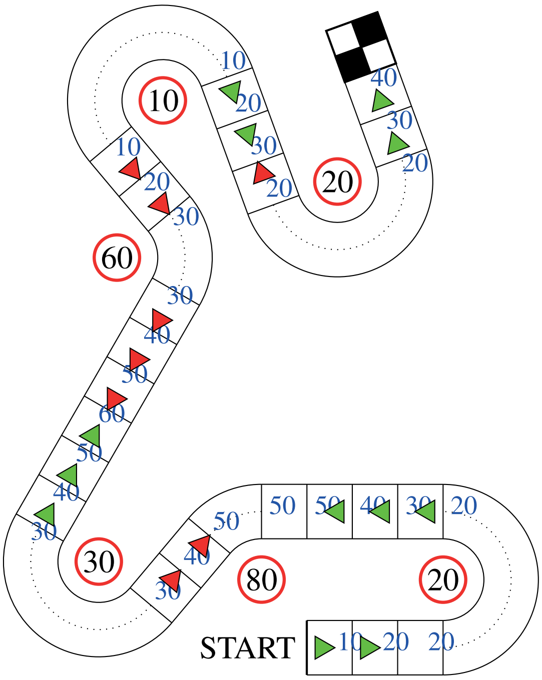
We address this as before by reducing the 70 to 50 and the 60 to 30.

The drive now becomes *start* 3 [20] 4 [50] 2 [30] 6 [30] 2 [10] 3 [20] 2 *end*.

We can now determine Alain's actions on each straight.

Straight	Drive
start 3 [20]	++∽
[30] 4 [50]	+++∽
[50] 2 [30]	−−
[30] 6 [30]	+++−−−
[30] 2 [10]	−−
[10] 3 [20]	++−
[20] 2 *end*	++

Cost $= 12 \times 10 + 8 = 128$.



Part C: Prize Question 1

Answers:

Prize 1. Race Track

A. 10

B. 14

Senior Solutions
Question 3

3. Bubble Up

On each dive, a large bubble can only move up one level (5 and 4 in the example). But a small bubble can move down several levels (2 in the example). This means that the number of dives is determined by how far *below* its final position each bubble starts.

bubble size	2	12	4	11	7	10	9	1	3	8	6	5
initial position	1	2	3	4	5	6	7	8	9	10	11	12
final position	11	1	9	2	6	3	4	12	10	5	7	8
distance below	-10	1	-6	2	-1	3	3	-4	-1	5	4	4

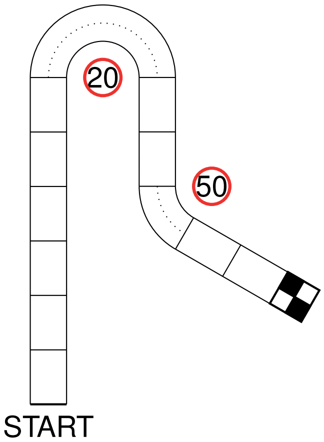
Bubbles 12, 11, 10, 9, 8, 6 and 5 are below their final positions.

The bubble that is most levels out of place is bubble 8. It starts 5 levels below its final position, so will require 5 dives to be put in place.
Hence (B).

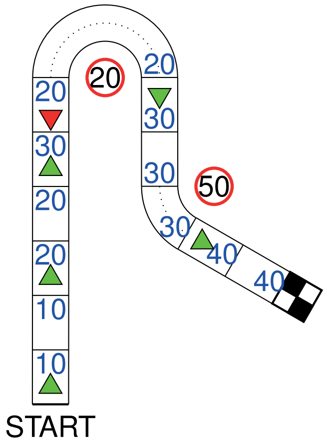
Part B: Question 7

7. Race Track

We will linearise the maps and use $+$ for accelerate, $-$ for brake and $\backsim$ for coast in the drives. Here is this notation applied to the example.



start 6 [20] 2 [50] 2 *end*



start $+ \backsim + \backsim + -$ [20] $+ \backsim$ [50] $+ \backsim$ *end*

When we know Alain's speed on each corner, his drive on each straight can be determined independently. Along a straight, his speed on each segment is greatest if he accelerates before cruising or braking, and cruises before braking.

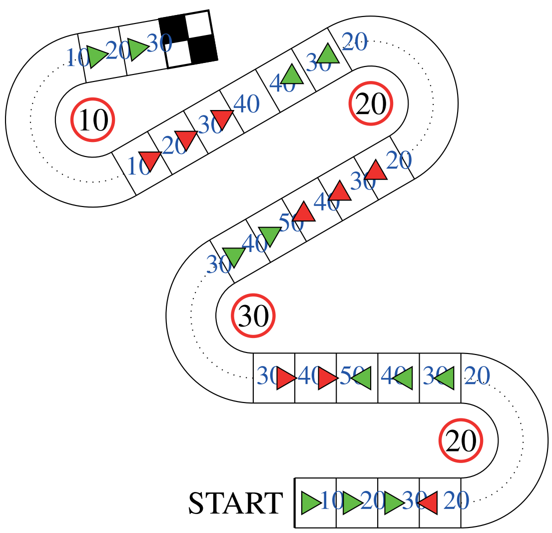
For track A, there are enough segments on the straight leading into a corner for him to take it at its limit.

A.

start 4 [20] 5 [30] 5 [20] 6 [10] 2 *end*

Straight	Drive
start 4 [20]	$+++-$
[20] 5 [30]	$+++--$
[30] 5 [20]	$++---$
[20] 6 [10]	$++\backsim---$
[10] 2 *end*	$++$

Cost $= 12 \times 10 + 9 = 129$.



B.

start 7 [60] 2 [30] 6 [60] 2 [10] 3 [20] 2 *end*

Here there are not enough segments to slow from 60 to 30 in the straight [60] 2 [30].

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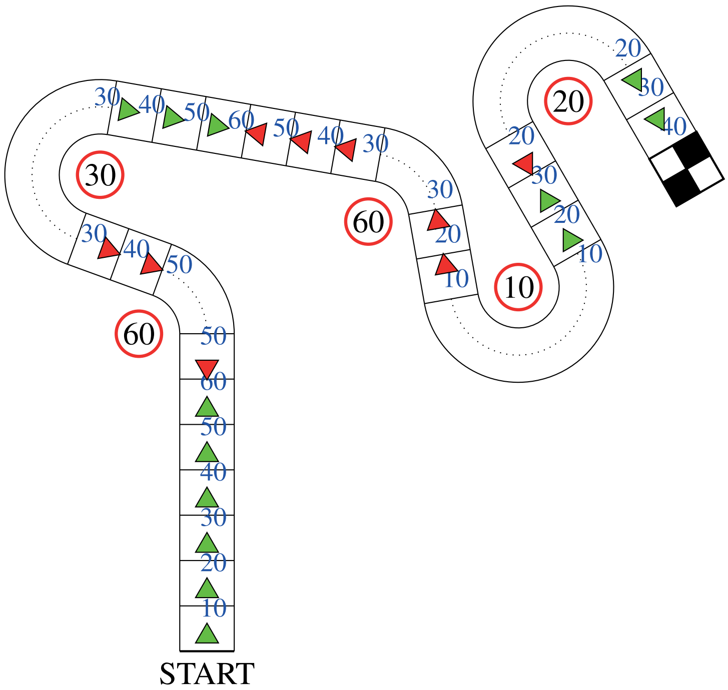
Straight	Drive
start 7 [50]	$+++++++-$
[50] 2 [30]	$--$
[30] 6 [30]	$+++---$
[30] 2 [10]	$--$
[10] 3 [20]	$++-$
[20] 2 *end*	$++$

Cost $= 13 \times 10 + 9 = 139$.



C.

start 3 [20] 4 [80] 2 [30] 6 [60] 2 [10] 3 [20] 2 *end*

On this track we have two problems. Firstly there are not enough segments on the second straight to reach the limit on the second corner.

Here we address this by reducing the speed on the second corner to 70.

The drive then becomes *start* 3 [20] 4 [70] 2 [30] 6 [60] 2 [10] 3 [20] 2 *end*.

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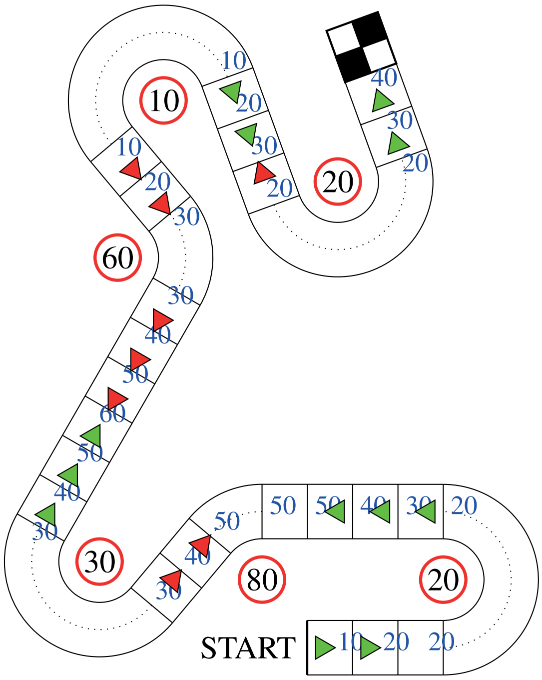
The drive now becomes *start* 3 [20] 4 [50] 2 [30] 6 [30] 2 [10] 3 [20] 2 *end*.

We can now determine Alain's actions on each straight.

Straight	Drive
start 3 [20]	++ ∽
[30] 4 [50]	+++ ∽
[50] 2 [30]	−−
[30] 6 [30]	+++−−−
[30] 2 [10]	−−
[10] 3 [20]	++−
[20] 2 *end*	++

Cost $= 12 \times 10 + 8 = 128$.



Part C: Prize Question 1

Answer:

Prize 1. Race Track

A. 10

B. 56