2024 Maths Challenge

Intermediate

Student Problems

AUSTRALIAN MATHS TRUST

MATHS CHALLENGE

I1 Pointy Numbers

A *pointy number* can be either upward pointy or downward pointy. An upward pointy number has digits in ascending order up to a digit called the *point* from which the remaining digits are in descending order. A downward pointy number has digits in descending order to a point digit from which the remaining digits are in ascending order. In all pointy numbers:

- If digits are adjacent, then they differ by 1.
- The first and last digits are the same and not 0.

For example:
- 34543 is an upward pointy number
- 434 is a downward pointy number
- 24642 is not a pointy number as adjacent digits differ by 2
- 654345 is not a pointy number as the first and last digits are different
- 789987 is not a pointy number since it does not have a single point
- 012343210 is not a pointy number as it starts and ends with 0.

a Explain why all 7-digit pointy numbers that are divisible by 3 must have their first digit divisible by 3.

b How many pointy numbers are divisible by 6?

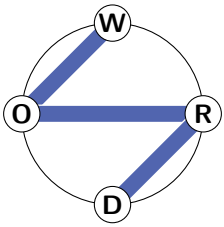
c An upward pointy number is added to a downward pointy number with the same number of digits. Show that the sum cannot be a prime number.

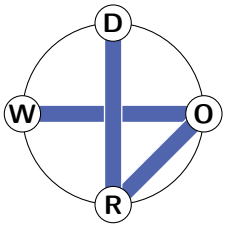
d How many pairs of 7-digit upward and downward pointy numbers when added give a sum that is a palindrome?

I4 Word Links

In the game *Word Links*, the letters of a word are placed around a circle in no particular order. The letters are then joined by links to indicate the word.

For example, here are two arrangements of the letters D, O, R, W with the links indicating the word WORD.





The arrangement on the left is called *crossless* because no two links cross each other. There are four crossless arrangements of the letters W, O, R, D to make the word WORD with W at the top.

a Draw the eight crossless arrangements for the word LINKS with L at the top.

b Find the probability that a random arrangement of the five letters in the word LINKS is crossless.

c Explain why there are 96 crossless arrangements for any word that has exactly six letters and the letters are all different.

d For words in which all letters are different, find the least number of letters for which the probability that a random arrangement is crossless is below 0.01.