2024 Maths Challenge

Junior

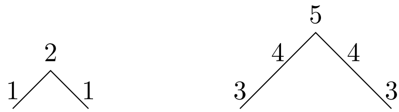
Student Problems

AUSTRALIAN MATHS TRUST

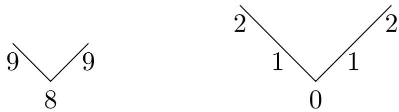
MATHS CHALLENGE

J1 Pointy Numbers

A *pointy number* can be either upward pointy or downward pointy. To create an *upward* pointy number, start with a single digit, write increasing consecutive digits up to a digit called the *point*, and then write decreasing consecutive digits back down to the original digit. Two examples of upward pointy numbers are 121 and 34543:



To create a *downward* pointy number, start with a single digit, write decreasing consecutive digits down to a digit called the *point*, and then write increasing consecutive digits back up to the original digit. Two examples of downward pointy numbers are 989 and 21012:



Pointy numbers must have more than one digit, and they cannot start with the digit 0. Here are some examples of numbers that are *not* pointy numbers:

- 7 is not a pointy number since it only has one digit
- 0123210 is not a pointy number since it starts with the digit 0
- 24642 is not a pointy number since its digits are not consecutive
- 654345 is not a pointy number since its first and last digits are different
- 789987 is not a pointy number since it does not have a single point.

a What is the largest pointy number that is divisible by 5?

b Explain why all 7-digit pointy numbers that are divisible by 3 must have their first digit divisible by 3.

c How many pointy numbers are divisible by 6?

d An upward pointy number is added to a downward pointy number with the same number of digits. Show that the sum cannot be a prime number.

J4 Card Fractions

Andy is playing a game using the following ten pairs of cards:

$$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{7}\ \boxed{8}\ \boxed{9}\ \boxed{10}$$

$$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{7}\ \boxed{8}\ \boxed{9}\ \boxed{10}$$

To play the card game, Andy chooses three pairs of cards (both cards in each pair have the same number) and uses them to make three *proper fractions*, that is, fractions that are less than 1. He then arranges the fractions in order from smallest to largest.

For example, if he chooses the pairs labelled 1, 2, 3, he makes the proper fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ because these are the only ones that are less than 1. He cannot make the fractions $\frac{3}{1}$, $\frac{2}{2}$ and so on. From smallest to largest, he arranges the proper fractions like so:

$$\frac{\boxed{1}}{\boxed{3}} < \frac{\boxed{1}}{\boxed{2}} < \frac{\boxed{2}}{\boxed{3}}$$

All cards are returned to the pile after each go.

a Andy chooses the card pairs labelled 4, 5, 8 and uses them to make three proper fractions. He then orders them from smallest to largest. Find the three fractions and their order.

b Andy chooses the pairs labelled 1 and 6 and one other pair. After arranging the proper fractions in order he flips over the third pair of cards:

$$\frac{\boxed{1}}{\boxed{6}} < \frac{\boxed{}}{\boxed{6}} < \frac{\boxed{1}}{\boxed{}}$$

Find all possibilities for the number on the pair of cards that has been flipped over.

c Andy chooses the pair labelled 7 and two other pairs. He arranges the proper fractions in order. Leaving the cards labelled 7 in place, he turns over one of the other pair of cards and writes the letter A on the back of each of those cards. He then turns over the remaining pair of cards and writes the letter B on the back of each of those cards. The result is shown:

$$\frac{\boxed{A}}{\boxed{B}} < \frac{\boxed{7}}{\boxed{B}} < \frac{\boxed{A}}{\boxed{7}}$$

Find all possible combinations for the numbers on the cards labelled A and B.

d Andy expands his collection to 2024 pairs of cards labelled 1 to 2024. He chooses the pairs labelled 1 and 2024, and one other pair. When the proper fractions are arranged in order, the largest fraction has 1 as the numerator. Find the number of possible values there are for the unknown number.