# 2024 Maths Challenge

## Upper Primary

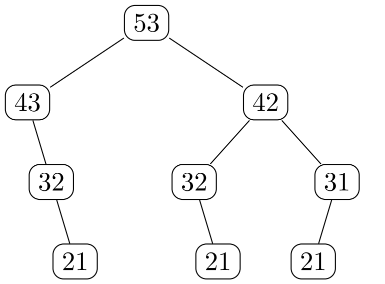
## Solutions for Students

AUSTRALIAN MATHS TRUST

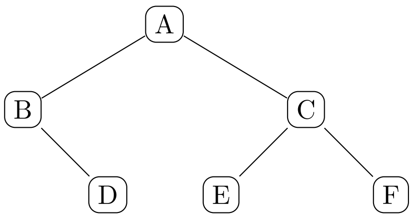
MATHS CHALLENGE

## UP2 Drop Bears

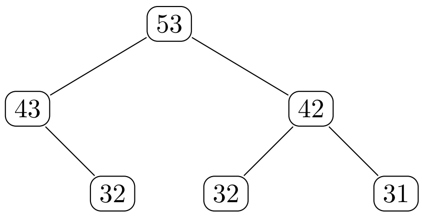
**a** Using drop bear subtraction, we get the following tree.



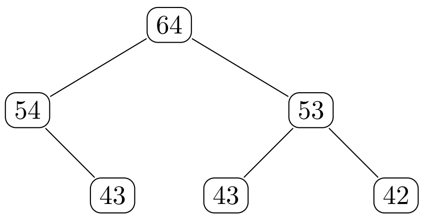

**b** We label the nests with the letters A to F as shown.




The only allowable nest below 43 is 32. So neither A nor C is 43. Hence one or more of B, D, E, F is 43. If B is 43, then we have the following partial tree.



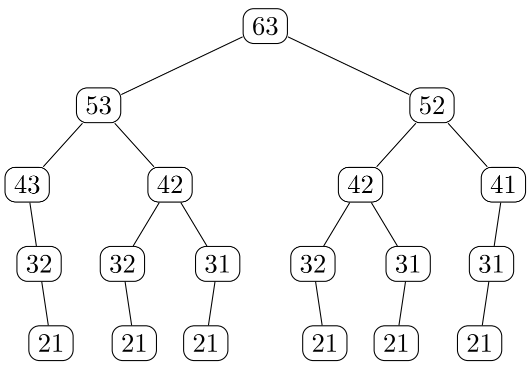

If D or E is 43, then we have the following partial tree.




*Comment*

If F is 43, then C is 54 and there is no address for E. So there are only two different partial trees.

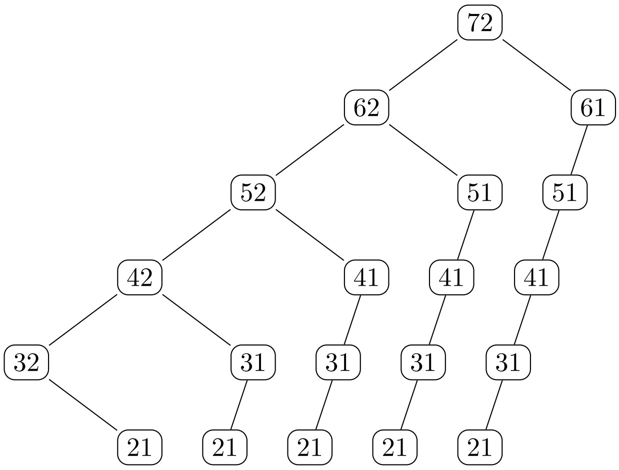
**c** From the example in the introduction and the solution to Part **a**, we see that drop bear tree 52 is the mirror image of drop bear tree 53 (ignoring the addresses). The common address above 52 and 53 is 63. So drop bear tree 63 is symmetrical, as seen below.



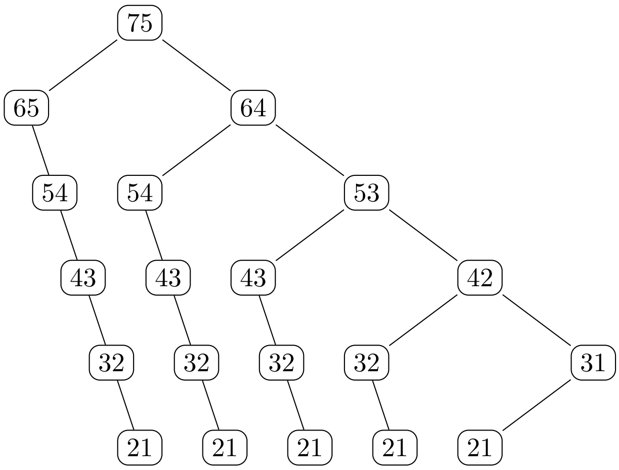

This tree may also be found by trial and error.

**d Alternative i**

Start by drawing drop bear tree 72. (Most of tree 72 is tree 62, which is in the introduction.)

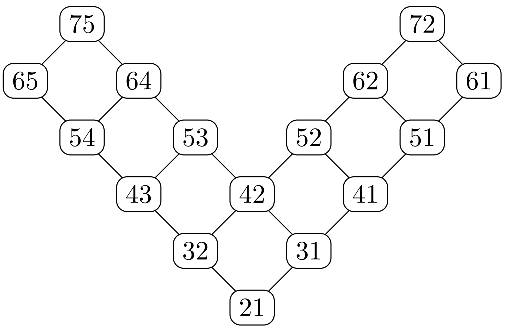



Now remove all nest addresses except for the 21s and then draw the mirror image of the tree. Working upwards from the bottom row, add addresses to the vacant nests. The result is drop bear tree 75.




**Alternative ii**

To have five separate nests with address 21 in a tree, there must be five different paths in the tree from home to those nests. To find those paths, we work upwards from 21 and place nests in a lattice as shown below, rather than a tree.




There are exactly five strictly downward paths from 72 to 21 and these form tree 72. There are exactly five strictly downward paths from 75 to 21 and these form tree 75. Both trees are displayed in the Alternative **i** solution.

*Comment*

Note that if any extra nest $N$ is added to the lattice, then tree $N$ will have exactly one nest 21 or it will have more than five nests 21.

## UP4 Card Fractions

**a Alternative i**

We use a common denominator. Since $\frac{2}{5} = \frac{14}{35}$, $\frac{2}{7} = \frac{10}{35}$, $\frac{5}{7} = \frac{25}{35}$, we have $\frac{2}{7} < \frac{2}{5} < \frac{5}{7}$.

**Alternative ii**

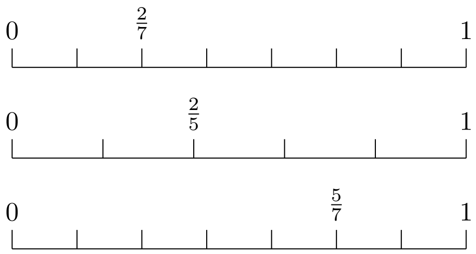
Converting each fraction to a decimal, we have $\frac{2}{5} = 0.4$, $\frac{2}{7} \approx 0.28$, $\frac{5}{7} \approx 0.71$. Thus $\frac{2}{7} < \frac{2}{5} < \frac{5}{7}$.

**Alternative iii**

We know that $\frac{2}{7}$ is less than $\frac{2}{5}$ because they have the same numerator and therefore the larger denominator gives the smaller fraction. Since $\frac{2}{5} < \frac{1}{2}$ and $\frac{5}{7} > \frac{1}{2}$, we have $\frac{2}{5} < \frac{5}{7}$. So $\frac{2}{7} < \frac{2}{5} < \frac{5}{7}$.

**Alternative iv**

We use number lines.




Thus $\frac{2}{7} < \frac{2}{5} < \frac{5}{7}$.

**b** Since the left fraction is less than the middle fraction, the cards Andy flipped must be labelled 3 or less.

Since the right fraction is greater than the middle fraction, which is greater than $\frac{1}{2}$, the flipped number must be 3 or more.

So the number on the cards Andy flipped must be 3.

**c** The cards Andy flipped cannot be labelled 1 since 1 is still visible.

If the cards Andy flipped were labelled 2, the fractions would have values $\frac{1}{6}$, $\frac{2}{6} = \frac{1}{3}$, $\frac{1}{2}$, which are indeed ordered from smallest to largest.

If the cards were labelled with a number greater than 2, the middle fraction would be at least $\frac{3}{6} = \frac{1}{2}$, while the fraction on the right would be at most $\frac{1}{3}$, hence not greater than the middle fraction.

So the only possible number on the cards Andy flipped is 2.

**d** Each fraction must be less than 1. So we know from the fraction on the right that the number on the A cards must be less than 7, and from the middle fraction that the number on the B cards must be greater than 7.

The table below summarises the possible outcomes for different A and B numbers. In each case, the fractions can be compared by any of the methods described in the solutions to Part **a**.

| B | A | Comparison | Outcome |
|---|---|---|---|
| 8 | 6 | $\frac{7}{8} > \frac{6}{7}$ | A too small. |
| 9 | 6 | $\frac{7}{9} < \frac{6}{7}$ | Possible. |
| 9 | $\leqslant 5$ | $\frac{7}{9} > \frac{A}{7}$ | A too small. |
| 10 | 6 | $\frac{7}{10} < \frac{6}{7}$ | Possible. |
| 10 | 5 | $\frac{7}{10} < \frac{5}{7}$ | Possible. |
| 10 | $\leqslant 4$ | $\frac{7}{10} > \frac{A}{7}$ | A too small. |

So the only possible combinations of numbers on the cards labelled A and B are respectively: 6 and 9, 6 and 10, 5 and 10.