2024 Maths Challenge

Upper Primary

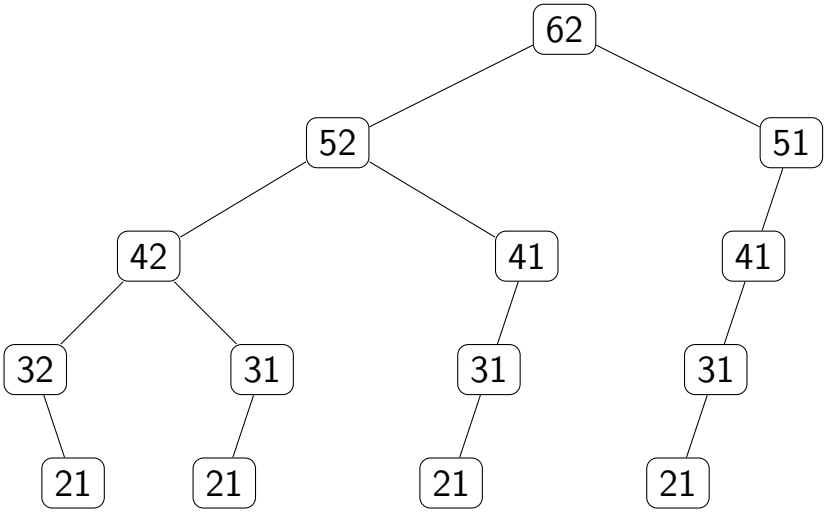
Student Problems

AUSTRALIAN MATHS TRUST

MATHS CHALLENGE

UP2 Drop Bears

Drop bears live in nests in drop bear trees. Each nest has a number. The nest with the largest number is called *home* and its number is used to name the tree. For example, the following diagram shows drop bear tree 62.



The nest numbers are called *addresses* and all have 2 digits. The first digit is always greater than the second digit and no digit is 0. For example:

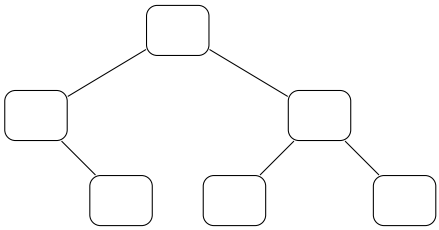
- 53 is an address since 5 is greater than 3
- 35 is not an address since 3 is not greater than 5

- 44 is not an address since the first digit is not greater than the second
- 50 is not an address since no digit can be 0.

Drop bears do a special subtraction to work out the number of their nest. Starting from home, to get the next address below and to the left they subtract 1 from the first digit. To get the next address below and to the right, they subtract 1 from both digits. Drop bear trees are always drawn until no more nests can be added.

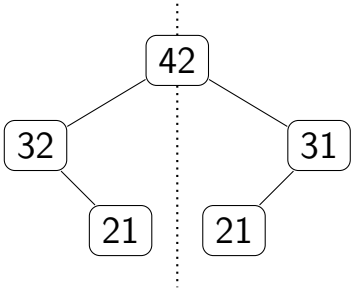
a Draw drop bear tree 53.

b This diagram shows part of a drop bear tree.



There are two different ways to fill in the addresses of the six blank nests so that at least one nest has address 43. Draw both ways.

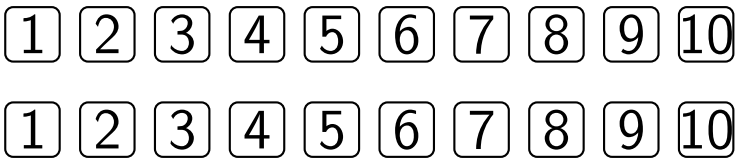
c Ignoring the addresses, drop bear tree 42 is symmetrical about the vertical line through home as shown. Find a symmetrical drop bear tree with more than five nests but fewer than 20 nests.



d Drop bear tree 72 has exactly five nests with address 21. Find another drop bear tree that has exactly five nests with address 21.

UP4 Card Fractions

Andy is playing a game using the following ten pairs of cards:



To play the card game, Andy chooses three pairs of cards (both cards in each pair have the same number) and uses them to make three *proper fractions*, that is, fractions that are less than 1. He then arranges the fractions in order from smallest to largest.

For example, if he chooses the pairs labelled 1, 2, 3, he makes the proper fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ because these are the only ones that are less than 1. He cannot make the fractions $\frac{3}{1}$, $\frac{2}{2}$ and so on. From smallest to largest, he arranges the proper fractions like so:

$$\frac{1}{3} < \frac{1}{2} < \frac{2}{3}$$

All cards are returned to the pile after each play.

a Andy chooses the pairs labelled 2, 5, 7 and makes the three proper fractions $\frac{2}{5}$, $\frac{2}{7}$, $\frac{5}{7}$. Arrange them in order from smallest to largest.

b Andy chooses the pairs labelled 4 and 6, and one other pair. After arranging the proper fractions in order, he flips over the third pair of cards as shown:

$$\frac{\square}{6} < \frac{4}{6} < \frac{\square}{4}$$

Find the number on the pair of cards that has been flipped over.

c Andy chooses the pairs labelled 1 and 6, and one other pair. As before, he arranges the proper fractions in order and flips over the third pair of cards:

$$\frac{1}{6} < \frac{\square}{6} < \frac{1}{\square}$$

Find all possibilities for the number on the pair of cards that has been flipped over.

d Andy chooses the pair labelled 7 and two other pairs. He arranges the proper fractions in order. Leaving the cards labelled 7 in place, he turns over one of the other pair of cards and writes the letter A on the back of each of those cards. He then turns over the remaining pair of cards and writes the letter B on the back of each of those cards. The result is shown:

$$\frac{\boxed{A}}{\boxed{B}} < \frac{\boxed{7}}{\boxed{B}} < \frac{\boxed{A}}{\boxed{7}}$$

Find all possible combinations for the numbers on the cards labelled A and B.